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Technical Notes

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Quadrilateral Plate Element for Laminated Structure

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SINCE the early work of Reissner and Stavsky[1] on laminated plates, the coupling phenomenon between in-plane stretching and transverse bending is known to be appreciable. This phenomenon does not occur in the homogeneous plate and was neglected by previous investigators. In 1962, Dong et al.[2] formulated the first general small deflection theory for the bending and extension of laminated anisotropic plates and shells. They considered the coupling between stretching and bending but neglected the transverse shear effect. However, recent studies[3-5] for laminated plates made with high-modulus fibrous composite materials show that the effect of transverse shear deformation is more prominent than for an isotropic homogeneous plate. More recently, Pryor and Barker[6] developed a rectangular plate element including the transverse shear effect for laminated structures.

In this paper a quadrilateral element is developed based on Pryor and Barker's work. The element is quite similar to, but more versatile than, the rectangular element developed by Pryor and Barker. Because it is quadrilateral in shape, the element can model curved boundary conditions better.

The quadrilateral element contains four nodal points and each node exhibits seven degrees of freedom: the translation displacements of the midsurface $\bar{u},\bar{v},w$; two rotations θ_x,θ_y due to bending; and two rotations γ_{xz},γ_{yz} due to transverse shear effect. The rotations θ_x,θ_y can be expressed as the slopes of the displacement w.

These displacement functions are assumed as follows:

$$\{f\} = [N]\{\delta^e\} \tag{1}$$

where

$$\{f\}' = [\bar{u}\bar{v}w\theta_y\theta_x\gamma_{xz}\gamma_{yz}]$$

matrix $[N]$ defines the nature of the interpolation functions, and $\{\delta^e\}$ is the array of the nodal generalized coordinates of the element.

Strains are expressed in terms of $\{\delta^e\}$ by using Eq. (1) and the strain-displacement relations for a moderately thick plate. The resulting strains are grouped into matrix form and symbolized as

$$\{\epsilon\} = [B]\{\delta^e\} \tag{2}$$

where

$$\{\epsilon\}' = [X_x X_y X_{xy} \bar{\epsilon}_x \bar{\epsilon}_y \bar{\epsilon}_{xy} \gamma_{xz} \gamma_{yz}]$$

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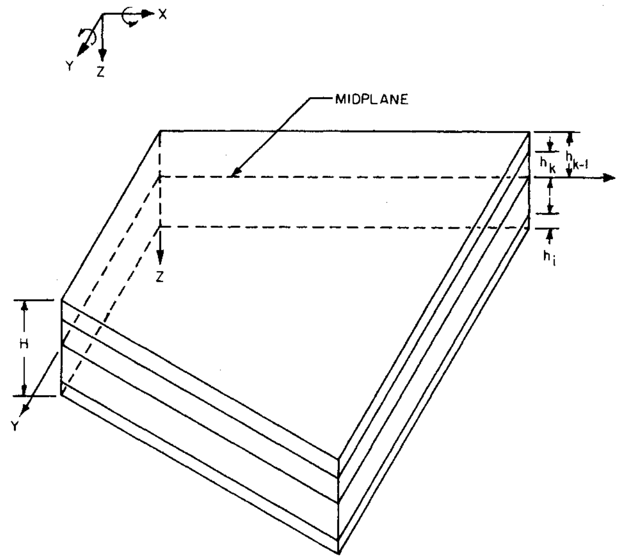


Fig. 1 Quadrilateral laminated plate element.

and

$$\bar{\epsilon}_x = \frac{\partial \bar{u}}{\partial x}, \quad \bar{\epsilon}_y = \frac{\partial \bar{v}}{\partial y}, \quad \bar{\epsilon}_{xy} = \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}$$

$$X_x = -\frac{\partial^2 w}{\partial x^2} + \frac{\partial \gamma_{xz}}{\partial x}$$

$$X_y = -\frac{\partial^2 w}{\partial y^2} + \frac{\partial \gamma_{yz}}{\partial y}$$

$$X_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{yz}}{\partial x}$$

The final form of the relation between the stress resultants and the midsurface strains and curvature can be written as

$$\{\sigma\} = [D]\{\epsilon\} \tag{3}$$

where

$$\{\sigma\}' = [m_x m_y m_{xy} n_x n_y n_{xy} q_x q_y]$$

is the generalized stress array. $[D]$ is a symmetric matrix of elastic constants that describe an anisotropic laminated material as given in Ref. 7.

Standard procedure can now be followed to obtain the element stiffness matrix and the element load vector. Since the element has a quadrilateral shape as shown in Fig. 1, isoparametric transformation is used to transform it into a parent square element, then numerical integration is applied to obtain the element stiffness and element load vector.

The preceding theory was then implemented in a finite-element computer program. The integration and assembly of the total stiffness matrices were then carried out numerically by the computer. In order to check the validity of the element, calculations were carried out for four examples.

1) A homogeneous simply supported square plate subjected to a uniformly distributed load as shown in Fig. 2. In order to compare the results with the Reissner theory,[6] a non-

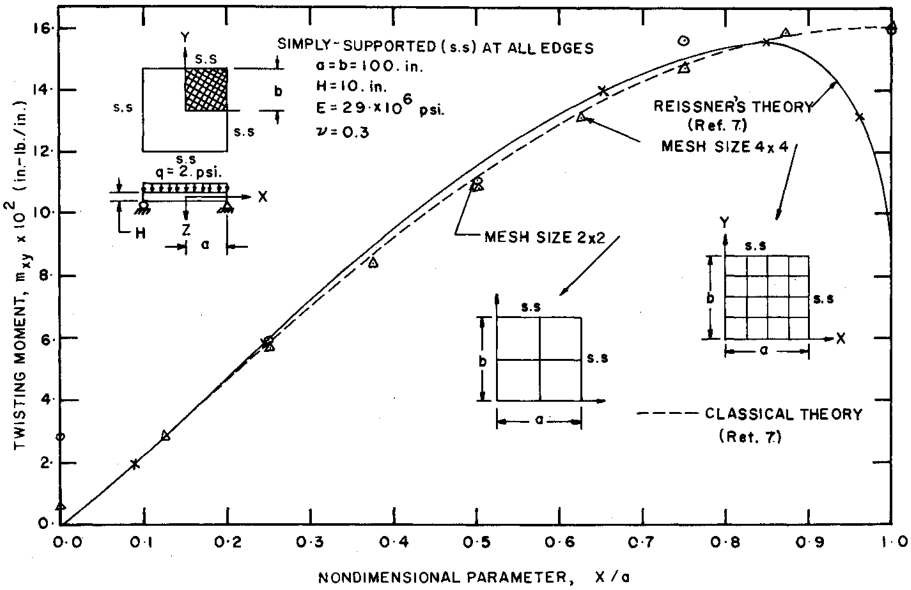
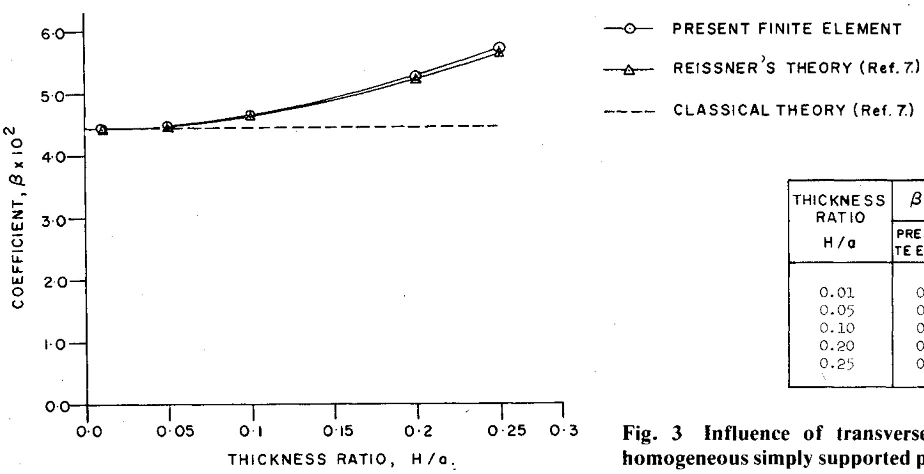


Fig. 2 Distribution of twisting moment m_{xy} along section $y = b/2$.



THICKNESS RATIO H/a	$\beta = W_{max}EH^3/qa^4$ UNIFORMLOAD, q PRESENT FINITE ELEMENT.	REISSNER'S THEORY	CLASSICAL THEORY
0.01	0.04481	0.04439	0.04437
0.05	0.04524	0.04486	0.04437
0.10	0.04686	0.04632	0.04437
0.20	0.05243	0.05217	0.04437
0.25	0.05698	0.05656	0.04437

Fig. 3 Influence of transverse shear on maximum deflection of a homogeneous simply supported plate.

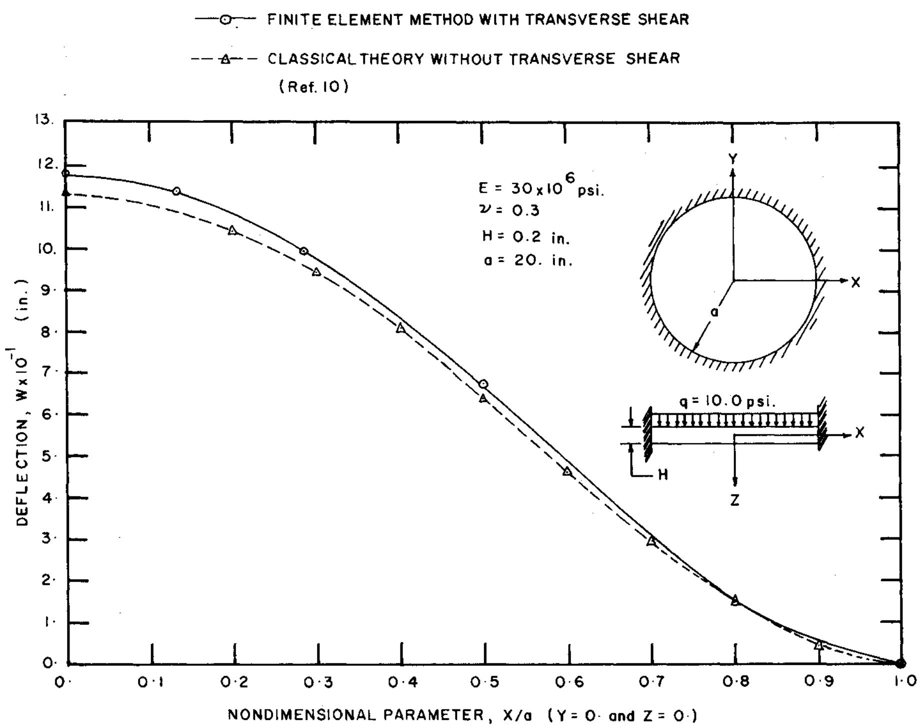


Fig. 4 Deflection of a homogeneous circular plate with built-in edge.

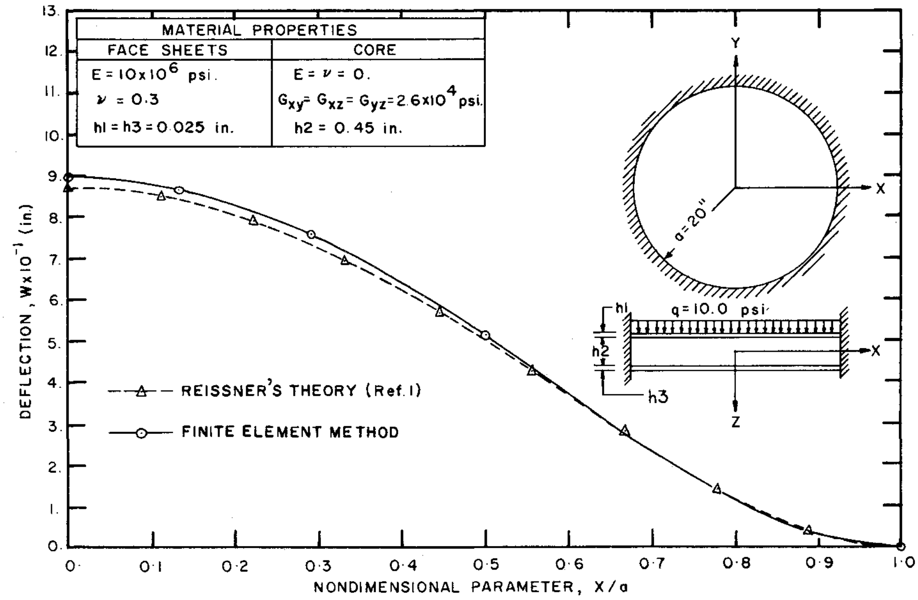


Fig. 5 Defection of a circular sandwich plate along section $Y=0$.

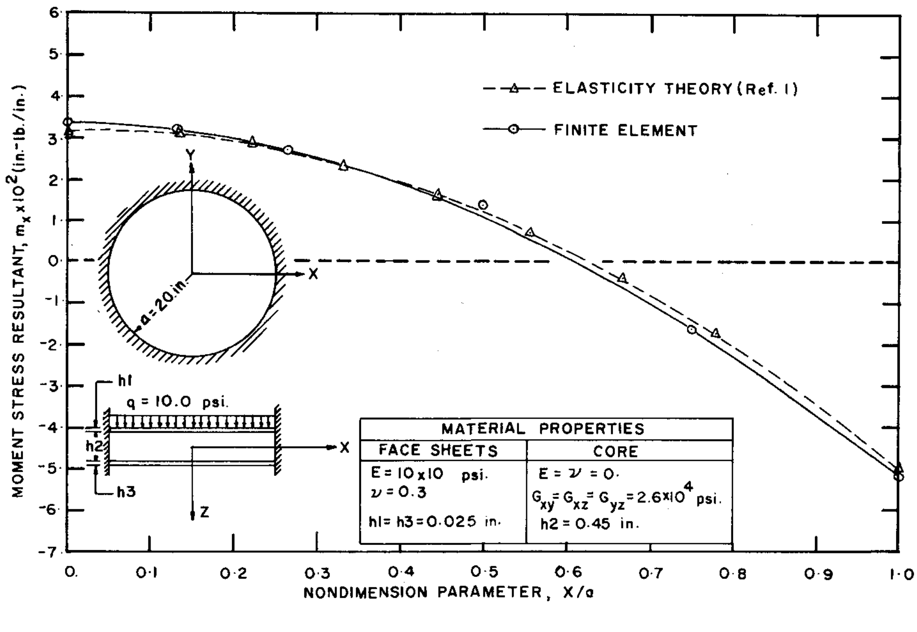


Fig. 6 Distribution of moment stress resultant m_x along section $Y=0$.

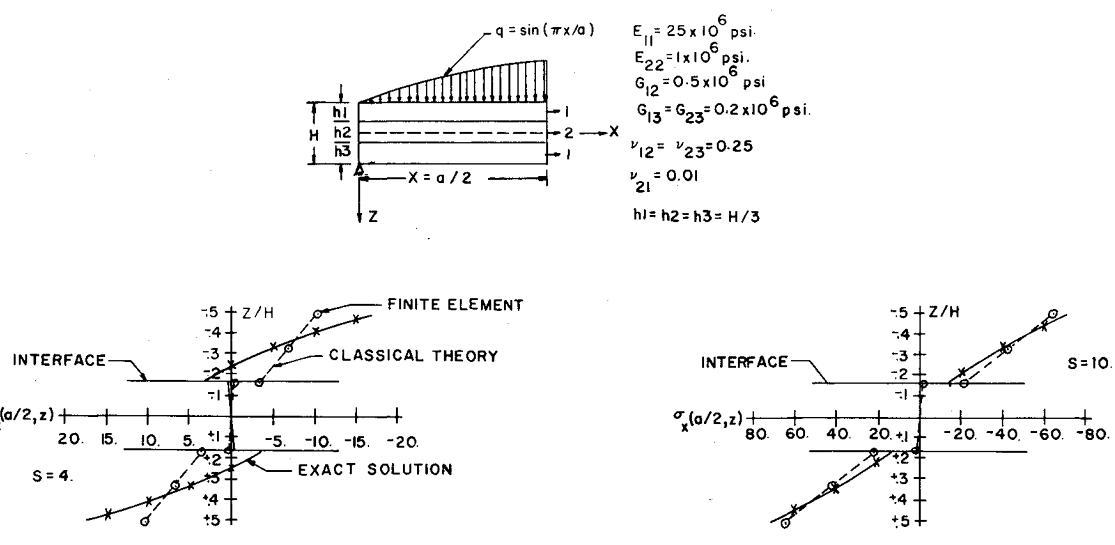


Fig. 7 Distribution of normal stress σ_x for a cylindrical bending of laminated composite plate.

dimensional deflection parameter was calculated for various plate thickness to lateral dimension ratios. These results are presented in Fig. 3. They show excellent agreement with the Reissner theory. Furthermore, the same example in Fig. 2 tested the limitation of the various mesh sizes. Results of the twisting moment m_{xy} seemed to agree with both the classical theory and Reissner theory as the mesh size decreased.

2) A homogeneous circular plate with built-in edge. Deflection was plotted against the nondimensional parameter x/a in Fig. 4. By comparing the present result with the classical theory,[7] an increase of maximum deflection due to transverse shear of 2.6% for the thickness ratio 0.01 was obtained, which appeared to be in satisfactory agreement with Reissner's theory.

3) A circular sandwich plate with built-in edge. The properties along with the results are shown in Figs. 5 and 6. The deflection w and the stress resultant m_x agree well with the results of the solution given by Ref. 8.

4) Cylindrical bending of a laminated composite plate subjected to a sinusoidal load. In Fig. 7 the coefficient β of the maximum deflection w was computed and compared with the value reported by Pagano,[7] excellent agreement was obtained.

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New Approach to the 3-D Transonic Flow Analysis Using the STAR-100 Computer

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I. Introduction

FINITE difference schemes have been successfully used to solve governing equations for transonic flow. The development of new algorithms in the early- and mid-1970s have lead to useful design tools for both 2-D airfoils[1,2] and 3-D wings.[3,4] However, because of the number of computations required and capacities of the current generation computers, only the 2-D flow analysis codes may be considered as cost effective. There is a great deal to be desired for the 3-D flow analysis codes. Before the supercomputer pursued by NASA Ames becomes operational, the newly matured vector processing machine STAR-100 shows strong potential of offering a wing designer the turn-around time that is so precious and sometimes vital. There has been only one document about solving the transonic flow equations on a vector machine.[5] The findings are somewhat disappointing. A new explicit scheme had to be developed in order to vectorize the computation. The convergence rate is approximately three times slower than the implicit successive line over-relaxation (SLOR).

This Note presents the findings of a Boeing team that investigated the potential of the STAR-100 computer in solving transonic flow problems. A scheme known to many numerical analysts as the 2-cyclic approach was adopted for solving the 3-D transonic small disturbance equation. The convergence rate was found to be no slower than the SLOR. Therefore, the speed advantage of STAR-100 over scalar machines such as CDC 6600 and 7600 can be realized.

II. Governing Equation and Solution Procedure

The transonic small disturbance equation

$$[(1-M_\infty^2)\phi_x - \tfrac{1}{2}M_\infty^2(\gamma+1)\phi_x^2]_x + [\phi_y]_y + [\phi_z]_z = 0$$

was written in the so-called conservative form. The Cartesian coordinate system was used to solve this equation numerically where y- and z-derivatives are always represented by central differencing. The x-derivatives are represented by either central differencing or backward differencing depending on the sign of $1-M_\infty^2-M_\infty^2(\gamma+1)\phi_x$. Classically, successive column relaxation has been used to solve the problem where only the ϕ values along a column of constant x,y coordinates, say, are solved simultaneously. All the values in the surrounding columns are considered to be known. The flowfield is swept column by column in an orderly fashion until convergence is achieved.

While the successive column relaxation has been used successfully for scalar machines, there are no easily identifiable vector type operations involved. Creating vector operations is first a programming consideration which in turn causes some modification to the numerical algorithm. In this case, the program that employs the SLOR had to be almost totally rewritten, but the modifications to the algorithm were relatively minor. The first attempt treated the whole y-z plane as a single vector, giving for this problem vector lengths of $M\times N$ for the generation of tridiagonal linear equations and M for the solution. M is the number of y meshes and N is the number of z meshes. This formulation altered the algorithm such that only ϕ values of previous iterations were used in the y-differencing as compared with mixed previous and current iteration values being used in SLOR. This resulted in a stability problem that prevented convergence in nonuniform mesh distributions. This difficulty gave rise to the use of the 2-cyclic method.

The 2-cyclic method splits the y-z plane into two sets of columns, the odd-numbered ones and the even-numbered ones. First, the standard column relaxation is carried out on all the odd numbered columns treated as a single vector. This cut the vector lengths to $(M\times N)/2$ for generation of equations and $M/2$ for solution. Velocity potentials obtained from the previous iterations were used to form y-derivatives for the odd-numbered columns. Then, the column relaxation is carried out on all the even-numbered columns treated as a single vector. The new ϕ values just obtained for the odd-numbered columns are used to form the y-differencings. This

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